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Passive Radiating Systems in Wave-Guides

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In a previous paper,[1] the writer gave the solution to the problem of the wave system produced in a wave-guide by a resonant slot, in which the voltage amplitude is in phase with the integral magnetomotive force. Here, we shall extend the results of that paper to the case of an arbitrary nonresonant radiating system consisting of passive metal antennas and apertures cut in the walls of the wave-guide.

Usually, in the theory of antennas, the electric currents in a single conductor, as well as the equivalent magnetic currents[2] in a single slot, are represented as a product of the geometric distribution function by the current amplitude, ascribing to the latter the characteristic resistance (impedance) of the antenna. We shall do exactly the same in our case for a complex system: we shall represent the electric currents $\mathbf{j}^e$ in the conductors and the equivalent magnetic currents $\mathbf{j}^m$ in the apertures as

$$\mathbf{j}_\alpha^e = I\mathbf{f}_\alpha; \qquad \mathbf{j}_\beta^m = I\mathbf{f}_\beta \tag{1}$$

(α being the indices of the conductors, and β the indices of the apertures), where $\mathbf{f}_\alpha$ and $\mathbf{f}_\beta$ are, generally speaking, the complex distribution functions of cm^{-2} dimensionality, and I is the "intensity of the current" in the system. (Actually, the equivalent magnetic currents in the apertures and the electric currents in the ideal conductors are surface and not volume currents. Therefore, the distribution functions f contain the noncharacteristic δ-functions, so that the integrals which contain f are reduced to surface integrals.) As the value I, we can take either the amplitude of the electric current in one of the conductors or the magnetic current in one of the slots or a value (somehow averaged) of these quantities for all the elements of the complex system—the choice of the "intensity of the current" in the system is dictated by the actual conditions of the problem. In any case, for a single radiator, I is simply a current amplitude.

Let us examine an infinite cylindrical wave-guide, in whose perfectly conducting walls apertures are cut, forming a passive radiating system with the conductors inside the wave-guide (Fig. 1). We normalize the characteristic waves of the pipe, which can be propagated inside it for a required working frequency in such a way that the average energy flow through the cross section of the wave-guide will be unity in each normalized wave. If we denote the transverse components of the field vectors in a normalized wave of the l type traveling in the +z-direction by

$$\mathbf{E}_l e^{-ih_l z}; \quad \mathbf{H}_l e^{-ih_l z} \tag{2}$$

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(hereafter, we shall always write only transverse components of the field vectors unless expressly stated otherwise; for more details on designations see (3)), then the conditions for normalization, along with the known property of orthogonality of waves of different types, are written as

$$\frac{c}{8\pi}\int [\mathbf{E}_l\mathbf{H}_n^*]_z dS = \delta_{ln}, \tag{3}$$

where δ_{ln} is Kroneker's symbol and the integral is taken along the cross section of the pipe. We note that for a normalized wave of the same type traveling in the $-z$-direction, the field vectors are

$$\mathbf{E}_l e^{ih_l z}; \quad -\mathbf{H}_l e^{ih_l z}. \tag{4}$$

Let us assume a wave of the pth type with the amplitude A traveling from the generator, which is at the end of the wave-guide, in the $+z$-direction [for the subscript "p," read "transverse"]:

$$\mathbf{E}_A = A\mathbf{E}_p e^{-ih_p z}; \ \mathbf{H}_A = A\mathbf{H}_p e^{-ih_p z}. \tag{5}$$

This primary wave (A-field) excites in the elements of the passive system electric and magnetic currents (1) which, in turn, are the sources of the secondary field (B-field). Inside the wave-guide outside the region occupied by the antenna, this secondary field is the resultant of the exponentially attenuated waves and the traveling waves of the type (4) or (2), in front or behind the antenna, respectively. Among the latter, there is also a wave of the same type as the primary field:

$$\mathbf{E} = b\mathbf{E}_p e^{-ih_p z}; \quad \mathbf{H} = b\mathbf{H}_p e^{-ih_p z}. \tag{6}$$

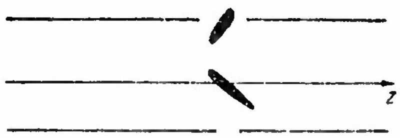

Fig. 1.

The waves in (5) and (6) mutually interfere and, therefore, in order to find a total wave of the pth type traveling in the $+z$-direction behind the antenna, we must know the modulus as well as the phase of the complex amplitude b (without destroying the generality, the amplitude A can be considered an actual magnitude). To determine the phase, we shall use the Lorentz complex lemma. In the general case of the two fields A and B, whose sources are the electric and the magnetic currents, this lemma in the differential form is described by the equation

$$-\frac{c}{8\pi}\operatorname{div}([\mathbf{E}_A\mathbf{H}_B^*] + [\mathbf{E}_B^*\mathbf{H}_A]) = \frac{1}{2}(\mathbf{E}_A\mathbf{j}_B^{e*} + \mathbf{E}_B^*\mathbf{j}_A^e) + \frac{1}{2}(\mathbf{H}_A\mathbf{j}_B^{m*} + \mathbf{H}_B^*\mathbf{j}_A^m), \tag{7}$$

which is a simple consequence of Maxwell's equations. In our case, the sources of the secondary field $\mathbf{j}_B^e$ and $\mathbf{j}_B^m$ are given by the formulas in (1); the sources of the primary field are

$$\mathbf{j}_A^e = 0; \quad \mathbf{j}_A^m = 0,$$

and the A-field itself is zero outside the wave-guide. Moreover, the tangential components of the electric vectors of both fields become zero on the walls of the pipe. Therefore, integrating (7) over the volume bounded by the walls of the pipe, by the surfaces closing the apertures from the outside, and by the two cross sections (in front and behind the antenna), we obtain, in view of the condition of orthogonality in (3), the equation

$$-2Ab^* = \frac{1}{2}\left(\sum_{\alpha}\int \mathbf{E}_A \mathbf{f}_\alpha^* d\,V + \sum_{\beta}\int \mathbf{H}_A \mathbf{f}_\beta^* d\,V\right) I^*, \tag{8}$$

where $\mathbf{E}_A$ and $\mathbf{H}_A$ are the total field vectors and not their transverse components, as in the preceding formulas.

The expression in parentheses is the integral "electromagneto-motive force" $\mathcal{E}^{em}$, which is applied to our passive system. The "intensity of the current" I mentioned previously is related to $\mathcal{E}^{em}$ by the linear formula $\mathcal{E}^{em} = ZI$, where

$$Z = R + iX = |Z|e^{i\varphi} \tag{9}$$

is the impedance (the characteristic resistance of the antenna). The actual part of the impedance, the radiation resistance R, is composed of the ohmic resistance of the metal elements of the antenna, of the resistance of the radiation into the outer space surrounding the wave-guide, and of the partial resistances of the radiation of characteristic waves of the pipe which travel from the antenna on both sides.

It follows directly from (8) and (9) that the complex amplitude b has the form $b = -|b|e^{-i\phi}$. As for the absolute value of this amplitude, for our normalization of the characteristic waves, it is related to the intensity of the current in the system by the obvious relation $|b|^2 = 1/2\tilde{R}|I|^2$, where $\tilde{R}$ is the partial resistance of the radiation of a wave of the pth type which travels in the $+z$-direction. Thus we have

$$b = -\sqrt{{}^1/_2\tilde{R}}\,|I|\,e^{-i\varphi}.$$

Substituting this expression in the equation for conservation of energy

$$|A + b|^2 + {}^1/_2(R - \tilde{R})|I|^2 = A^2$$

and solving the latter, we find

$$|I| = \frac{A\sqrt{8\tilde{R}}\cos\varphi}{R} = \frac{A\sqrt{8\tilde{R}}}{|Z|}, \tag{10}$$

$$b = -\frac{2A\tilde{R}e^{-i\varphi}}{|Z|} = -\frac{2A\tilde{R}}{Z}.$$

Therefore, the coefficient of transmission (over the field) G and the energy coefficient of transmission Q for a wave of the pth type are

$$G = \frac{A+b}{A} = \frac{Z - 2\tilde{R}}{Z}; \quad Q = |G|^2 = \left|\frac{Z - 2\tilde{R}}{Z}\right|^2. \tag{11}$$

The energy carried off by any other wave of the pipe (particularly by a wave of the pth type traveling in the $-z$-direction) or the energy radiated outward from the wave-guide is computed from the formula $W = 1/2R'|I|^2$, where R' is the corresponding radiation resistance. By means of (10), it is easy to find the corresponding energy coefficient of transformation:

$$Q' = \frac{W}{A^2} = \frac{4\tilde{R}R'}{|Z|^2}. \tag{12}$$

Eqs. (11) and (12) describe the wave system produced in the passively radiating system in an infinite wave-guide.

The problem of a semi-infinite wave-guide with a reflecting bottom, in the vicinity of which is a passive antenna, is solved in a similar way. The formulas found in this case have exactly the same form as those in (11) and (12) with the only obvious difference that the formulas in (11) will not give the transmission coefficient but the reflection coefficient.

In references 1 and 4, we give the solution of the problem of the rotation of the plane of symmetry of the wave produced by the resonant slot cut in a wave-guide having a round cross section. If the slot is not a resonant slot (i.e., if Z is complex), then the symmetrical and antisymmetrical waves behind the slot are no longer found in phase, and therefore the sum is an elliptically polarized wave. For the resonant slot, on the other hand, this sum is the usual linearly polarized wave, but with a slightly rotated plane of symmetry.

[1] M. L. Levin, Izvest. Akad. Nauk SSSR, ser. fiz., 12, 310 (1948).
[2] M. L. Levin, Zhur. tekh. fiz., 21, 787 (1951).
[3] M. L. Levin, Doklady Akad. Nauk SSSR, 60, 787 (1948).
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